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The Relationship between Kernel Set and Separation via ω-Open Set

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Abstract: **The main aim of this work to study and discuss some of separation axiom and the relation with the kernel set by using ω -open set.**

Keywords: **Separation axiom, ω - open set, ω -T_0 space, ω -T_1 space and Λ_ω (A).**

1. INTRODUCTION

This paper deals with separation axioms by using kernel set of a topological space with concept ω -open set. In1943, Shanan ,N,A.[10] define the concept of Called A subset A of a space

$\mathcal{K}$ *is called* $\omega - set$ if A = U ∩ V When U is open and int(V) = $int_{\omega(V)}$. In 1982 Hdeib , H. Z.[4] define the concept of $[\omega - open, \omega - interior\ and\ \omega - closure]$ where a subset W of a space $(\mathcal{K}, \mathfrak{T})$ is $\omega - open$ if and only if $\forall\ k \in W, \exists U \in \mathfrak{T} \ni k \epsilon U\ and\ U/W\ is\ countable$. We denoted the collection of all $\omega - open$ in $\mathcal{K}$ by ω.O($\mathcal{K}$) *and* also we denoted the collection of all $\omega - closed$ subset of $\mathcal{K}$ by ω.C ($\mathcal{K}$) and the union of all $\omega - open$ sets contained in A is called $\omega - interior\ of\ A$ and denoted by $int_{\omega(A)}$. also the collection of all $\omega - closed$ sets containing A is called $\omega - closure$ of A and denoted by $cl_{\omega(A)}$. In 2007 Al-omari ,A .and Noorani, M.S.M.[1] used the $\omega - open$ set to define the concept of $\omega - space$:- A topological space $(\mathcal{K}, \mathfrak{T})$ is called $\omega - space$ if every $\omega - open\ set$ is open in$\mathcal{K}$ *..In 2009 Noiri ,T., Al − Omari, A.A. and Noorain* , M ,S .M[7] define the concept : *Let* $(\mathcal{K}, \mathfrak{T})$ *be a topological space. It said to be satisfy*:.

1) The $\omega - condition\ if\ every\ \omega - open\ is\ \omega - set$.

2) The$\omega - \alpha\beta - condition\ \ if\ every\ \alpha - \omega - open\ set\ is\ \omega - \beta\alpha - set$.

3)*The* $\omega - B - condition\ if\ every\ pre - \omega - open\ is\ \omega - B - set$.

In 1977 Sharma .J.N [9] define the concept of T1-space: - A topological space $(\mathcal{K}, \mathfrak{T})$ is a T1-space if and only if every singleton {k} of $\mathcal{K}$ is closed. In 2011 Kim ,Y .k ,Devi ,R .and Selvarakumar, [5] define the concept of ultra separated :- A set A is said to be ultra separated from B if ∃ open set G ∋ A⊂G and G∩ $B = \phi$ or A∩ cl(B) = ϕ. *This paper consists of three section. In the first section we recall some* of the basic definitions that are connected with this research .In the second section

We prove some theorems, proposition about the concept of separated . *In the last* section we study a new type of of separation axiom as well as link the term separated with the classical separation axiom and we prove some theorems and properties about the concept.

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2. SEPARATED AND KERNEL SET

You will know a new type of separated also kernel set by using w-open and get some properties that will need in separation axiom.

Defintion2.1:-A set A in a topological space $(\mathcal{K},\mathfrak{T})$)is said to be:-

1) Weakly ultra ω_1 –separated (denoted by $\omega\omega_1$-sep.) from B if ∃ open set G such that G∩B≠ ϕ or A∩cl_ω (B)≠ ϕ.

2) Weakly ultra ω_2-separated (denoted by $\omega\omega_2$-sep.) from B if ∃ ω −open set G ∋ G∩B≠ ϕ or A∩cl_ω (B)≠ϕ.

3) Weakly ultra ω_3 -separated (denoted by $\omega\omega_3$-sep.) if ∃ ω-open set G ∋ G∩B≠ϕ or A∩cl(B)≠ϕ .

Proposition2.2:- In a topological space $(\mathcal{K},\mathfrak{T},)$then the following ststment are holds:-

1) If A is $\omega\omega_1$-sep.from B,then A is$\omega\omega_2$-sep. from B.

2) If A is $\omega\omega_2$-sep.from B,then A is$\omega\omega_3$-sep .from B.

Proof (1) :- Let $(\mathcal{K},\mathfrak{T})$) be a topological space and A be $\omega\,\omega_1$ − sep. from B ,then ∃ open set G *such* that G∩B≠ϕ or A∩clω (B)≠ϕ
by Lemma 1.2.4[4] we have ω -open set G such that G∩B≠ϕ or A∩cl_ω (B)≠ϕ by Definition2.1we get that A is $\omega\omega_2$-sep .from B.

(2):-it is clear.

Corolory2.3:- In a topological space $(\mathcal{K},\mathfrak{T})$) if A is $\omega\omega_1$-sep .from B , then A is$\omega\omega_3$-sep. from B.

Proof:- Let A is $\omega\omega_1$-sep . from B ,Then ∃ open set G∋ G∩B≠ϕ or A ∩clω (B)≠ ϕ by Lemma 1.2.4[4] we have ω- open set G∋ G∩B≠ ϕ or A∩cl(B)≠ϕ.then by Definition2.1we get every $\omega\omega_1$-sep.is$\omega\omega_3$ -sep.

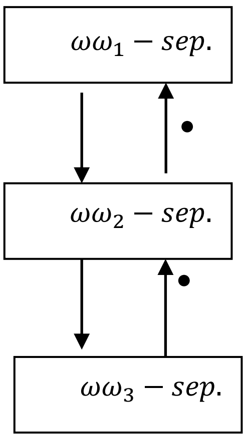
Proposition2.4:- Let$(\mathcal{K},\mathfrak{T})$) be a topological space with ω-condition ,then the following statement are hold:-

1) If A is $\omega\omega_3$-sep .from B , then A is $\omega\omega_2$-sep .from B.

2) If A is $\omega\omega_2$sep .from B , then A is $\omega\omega_1$-sep .from B.

Proof (1):- Let A be $\omega\omega_3$-sep .from B ,so∃ -open set G such that G∩B≠ϕ or A∩cl(B)≠ϕ. Since $\mathcal{K}$ is satisfy the ω -condition and by Lemma 1.2.4[4] we have cl(B)=cl_ω (B),implies that if A ∩cl (B)≠ϕ ,so A ∩cl_ω (B)≠ ϕ.Hence A is $\omega\omega_2$-sep. from B. if G∩B≠ϕ,Then also A is $\omega\omega_2$-sep.from B.

Proof (2): it is clear.



• $\omega - condition$

This figure show the relation between types separated with respect to ω-open.

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Defintion2.5:-For a subset A of a topological space $(\mathcal{K},\mathfrak{T})$),asubset $\Lambda_\omega(A)$=∩ {U:Asubset of U:U$\epsilon\omega$. **(1** o$(\mathcal{K},\mathfrak{T})$)}.

Lemma2.6:-For a subset A,B and A_i (i∈Δ) of a topological space $(\mathcal{K},\mathfrak{T})$),the following properties are holds:-

1)A ⊂ Λ_ω(A).

2)If A ⊂ B,then $\Lambda_\omega(A) \subset \Lambda_\omega(B)$.

3) $\Lambda_\omega(\Lambda_\omega(A) = \Lambda_\omega(A)$.

4)If A ∈ $\omega.o(\mathcal{K},\mathfrak{T}), then\ A = \Lambda_\omega(A)$.

5)$\Lambda_\omega\{\cup A_i: i\epsilon\Delta\}$ =∪ $\{\Lambda_\omega(A_i): i\epsilon\Delta\}$.

6)$\Lambda_\omega\{\cap A_i: i\epsilon\Delta\}$ ⊂∩ $\{\Lambda_\omega(\mathrm{A_i}): \mathrm{i}\epsilon\Delta\}$.

According to (1,3,4,5,6)the proof is clear from Defintion2.5

So will proof only (2)

Let k $\epsilon\ \Lambda_\omega(A), so\ k\epsilon \cap \{U: A \subset U: U\epsilon\omega.o(\mathcal{K})\}$

But A⊂ B⊂ U,thus kϵ U∀ U€ω.o($\mathcal{K}$) and B ⊂ U.

There for k∈∩{U:Uϵ.o($\mathcal{K}$) and B ⊂U}=Λ_ω (B). Hence k$\epsilon\Lambda_\omega$ (B). .

Defintion2.7:- A subset A of topological space $(\mathcal{K},\mathfrak{T})$is $\Lambda - \omega -$set if A=$\Lambda_\omega(A)$.

Lemma2.8:- A subset A and $A_i(i \in\Delta)$of a topological space $(\mathcal{K},\mathfrak{T})$,the following properties are holdes

1) $\Lambda_\omega(A) is\ \Lambda - \omega - set$

2)If A is an ω-open ,then Ais $\Lambda - \omega - set$

3)If Ai is $\Lambda - \omega - set$∀ i∈Δ,then∪ {A_i:i∈Δ}is $\Lambda - \omega - set$.

4)If A is $\Lambda - \omega - set$∀ i∈Δ,then ∩{A_i:i∈ Δ}is$\Lambda - \omega - set$.

According to(1,3,4)the proof is clear by Defintion2.7so will proof only (2)

Let A is ω-open set ,so by Lemma2.6 part(2) we get A=$\Lambda_\omega(A)$ then by Defintion2.7 A is $\Lambda - \omega - set$.

3. SEPARATIONS AXIOMS

We will use the $\boldsymbol{\omega}$-open to define the new type of separation axiom as well as link the term separated with the classical separation axiom and the conclusion of some properties and link kernel set with separated.

Defintion3.1:- $\mathcal{H}$is ω-Hausdorf in$\mathcal{K}$ if for two distinct point k and h of$\mathcal{H}$,there are disjoint ω-open subset U and V of $\mathcal{K} \ni k\epsilon U, h\epsilon V$.

Defintion3.2:-

$\mathcal{H}$is ω-regular in $\mathcal{K}$if ∀h of $\mathcal{H}$ and ∀closed subset p of $\mathcal{K}$ ∋ h∉ p,there are disjoint$\omega - open\ subset\ U\ and\ V\ of\ \mathcal{K} \ni h\epsilon U\ and\ p \cap \mathcal{H}\ subset\ of\ V$.

Defintion:-3.3 $\mathcal{H}$ is ω-supper regular in $\mathcal{K}$if $\forall h\ of\ \mathcal{H} and\ each$ closed subset P of $\mathcal{K} \ni h \notin p\ there\ are\ disjoint\ \omega - open\ subset\ U\ and\ V\ of\ \mathcal{K} \ni h \in U, p \subset V$.

Defintion3.4:-$\mathcal{H}$ is strongly ω-regular in $\mathcal{K}$:if $\forall k\ of\ \mathcal{K}$and each closed subset p of $\mathcal{K}$,there are disjoint ω-open subset U and V of $\mathcal{K}$ ∋ k∈ $U$and p∩ $k\ \subset V$.

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Defintion3.5:-A subset A of a topological space($(\mathcal{K},\mathfrak{T})$) is called$\omega$ -dense if ω-clA=$\mathcal{K}$.

Theorem3.6:-Let $(\mathcal{K},\mathfrak{T})$) be a topological space a subset $\mathcal{H}$ of $\mathcal{K}$ is ω-dense if and only if ∀ U∈ $\omega.o(\mathcal{K})$∋ U∩ $\mathcal{H} \neq \phi$

Proof :-suppose that $\mathcal{H} is$ ω-dence $\Leftrightarrow \omega - cl\mathcal{H} = \mathcal{K}$.

$\Leftrightarrow$ ∀k∈ $\mathcal{K}$and $\forall k \in U \in \omega.o(\mathcal{K}) \ni U \cap \mathcal{H} \neq \phi$.

$\Leftrightarrow$ ∀ U∈ $\omega.o(\mathcal{K}) we\ have\ U \cap \mathcal{H} \neq \phi$.

Theorem3.7:- If $\mathcal{H}$is a ω-dence subspace of a space $\mathcal{K}$ .Then $\mathcal{H}$is ω-Hausdorf in $\mathcal{K}$if and only if $\mathcal{H}$is ω-Hausdorf.
Proof:- → Let $h_1\ and\ h_2$ are arbitrary distinct points of$\mathcal{H}$. Since $\mathcal{H}$is ω-Hausdorf in$\mathcal{K}$,so ∃two disjoint ω-open subset$U_1$and V_1 in $\mathcal{K} \ni h_1 \in U_1$,$h_2 \in V_1$.We may assume that U=$U_1 \cap h$ and$V = V_1 \cap h$ .Then U and V are two ω-open set in $\mathcal{H}$and ∋:U∩ $V = \phi$,$h_1 \epsilon U\ and\ h_2 \epsilon V$.That is $\mathcal{H}$ is ω-Hausdorf.

← Let $h_1 and\ h_2$ are arbitrary distinct points of $\mathcal{H}$ .Since $\mathcal{H}$is ω-Hausdorf ,so ∃two disjoint ω-open subset $U_1 and\ V_1$in$\mathcal{H}$,∋ $h_1 \in U_1\ and\ h_2 \in V_1$ .So ∃two ω-open set U and Vin $\mathcal{K} \ni U_1$ = U∩ h and V_1=V∩ h, as follows we will prove U∩ $V = \phi$.

We may also assume that U∩ $V \neq \phi$.Then U∩ $V$is open in $\mathcal{K}$ since U and V are ω-open in $\mathcal{K}$,and $\mathcal{H}$is w-dence subspace of$\mathcal{K}$,so (U∩ V)∩$\mathcal{H} \neq \phi$.therfor$U_1 \cap V_1$,this is contradict with $U_1 \cap V_{1=}\phi$,so U∩ $V = \phi$.That is $\mathcal{H}$ is ω-Hausdorf in$\mathcal{K}$.

Theorem3.8:-If $\mathcal{H}$ is ω-closed (open) subspace of$\mathcal{K}$.then $\mathcal{H}$ is ω-regular in $\mathcal{K}$ if and only if $\mathcal{H}$is ω-supper regular in$\mathcal{K}$.

Proof:-→Let h is an arbitrary point of $\mathcal{H}$and an arbitrary closed subset P of $\mathcal{K}$,∋ $h \notin P$.since $\mathcal{H}$is ω-regular in$\mathcal{K}$,there are disjoints ω-open subset $U_1 and\ V_1$ of $\mathcal{K}$

∋ $h \in U_1$ and p∩$\mathcal{H} \subset V_1$.We assume that U=$\mathcal{K} \cap U_1$1,then since h is a point of $\mathcal{H}$and $\mathcal{H}$is ω-open in$\mathcal{K}$,so $\mathcal{K} / \mathcal{H}$is an ω-open set of$\mathcal{K}$.We may also assume that V=$V_1 \cup (\mathcal{K} / \mathcal{H})$,then we can get :p⊂V and U∩ $V = \phi$.That is $\mathcal{H}$is ω-supper regular in$\mathcal{K}$.

←Let h is an arbitrary point of and an arbitrary closed subset P of$\mathcal{K}$,∋ $h \notin P$

.there are disjoint ω-open subset U and V of $\mathcal{K} \ni h \in U$ and P⊂ V.obviously ,P∩$\mathcal{H}$subset of V that is $\mathcal{H}$is w-regular in$\mathcal{K}$.

Defintion3.9[6]:-Let$\boldsymbol{\mathcal{K}}$ be a topological space if for each k≠ $\boldsymbol{h} \in \boldsymbol{\mathcal{K}}$,either a set U ∋k$\boldsymbol{\epsilon U}$,h∉ $\boldsymbol{U}$,or there exist a set V∋ $\boldsymbol{h} \in \boldsymbol{V}, \boldsymbol{k} \notin \boldsymbol{V}$.Then $\boldsymbol{\mathcal{K}}$is called $\boldsymbol{\omega - T_0}$ space whenever U is $\boldsymbol{\omega - open}$ set in $\boldsymbol{\mathcal{K}}$.

Defintion3.10[6]:-Let$\mathcal{K}$ be a topological space if for each k≠ $h \in \mathcal{K}$.∃ a set U∋ $k \in U, h \notin U and \ni a\ set\ V \ni h \in V, k \notin V, then\ \mathcal{K}\ is\ called\ \omega - T_1 space\ if\ U\ is\ open\ and\ V\ is\ \omega - open\ set\ in\ \mathcal{K}$.

Theorem3.11:-If a topological space $(\mathcal{K},\mathfrak{T})is\omega - T_0 space$,then either $\{k\}$is

$\omega\omega i - sep.from$\{h }or {h }is $\omega\omega i$-sep.from {k },when i=2,3

Proof:-

Let$(\mathcal{K},\mathfrak{T})$be $\omega - T_0 space \forall k \neq h \in \mathcal{K}$,∃$\omega -$ -open set G
$\ni k \in G, h \notin G, or\ h \in G, k \notin G, then\ either\ \{k\} is\ not\ \omega\omega i.sep.from\ \{h\} or\ \{h\} is\ not\ \omega\omega i - sep.from\ \{k\}$.

Theorem3.12:-

Let $(\mathcal{K},\mathfrak{T})\ be\ \omega - T_1 space$ then either {k }is $\omega\omega i$-sep.from {h }or {h }is $\omega\omega i$-sep.from {k }∀ k≠ $h \in \mathcal{K}$,when i=2,3.

Proof:-Let $(\mathcal{K},\mathfrak{T})$,be $\omega - T_1 space, then\ \forall k \neq h \in \mathcal{K}$, ∃ω -open set U,V∋ k∈U,h∉ $U\ and\ h \in V, k \notin V$.
.implies {k }⊂ $\omega -$ open set U,{h }∩U=ϕthen{k }is$\omega\omega i$ -sep. from{h }and {h } ⊂ $\omega -$ open set V,{k }∩V=ϕ.then {h }is $\omega\omega i - sep.from\ \{k\}$.

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is$\omega\omega i - sep.$ **Thorem3.13**:-Let $(\mathcal{K},\mathfrak{T})$be a topological space if $\Lambda_\omega\{k\}$

from$\{h\} or\ \Lambda_\omega\{k\} is\ \omega\omega i - sep. from\{k\} \forall k \neq h \in \mathcal{K}, iff\ (\mathcal{K},\mathfrak{T}) is\ \omega - T_0 space\ , when\ i = 2,3.$

Proof:-Let either $\Lambda_\omega\{k\}$ *is* $\omega\omega i$-sep .from {h }or $\Lambda_\omega\{h\}$ be $\omega\omega i$-sep .from{k },when i=2,3 iff $\exists\omega$ –-open set G such that $\Lambda_\omega\{k\}$ ⊂Gand G∩{h }=ϕor $\Lambda_\omega\{h\}$ ⊂ Gand G∩{k} =ϕ.↔ω -open set G containing h but not k or k but not h .thus the topological .space is$\omega - T_0 space$..

Thorem3.14:-Let $(\mathcal{K},\mathfrak{T})$ be a topological space from {h }if $\Lambda_\omega\{k\}$}is $\omega\omega i$ sep.from{h }and $\Lambda_\omega\{h\}$is $\omega\omega i$-sep.from {k },$\forall k \neq h \in \mathcal{K}$, iff $(\mathcal{K},\mathfrak{T})$ is $\omega - T_1$space ,when i=2,3.

Proof:-Since $\Lambda_\omega\{k\} is\ \omega\omega i$ –sep.from {h }, then ∃ ω-open set G ∋ $\Lambda_\omega\{k\}$⊂ Gand G∩{h }=ϕ.and $\Lambda_\omega\{h\}$is$\omega\omega i$ -sep. from{k },then ∃ an ω-open set F∋ $\Lambda_\omega\{h\}$⊂ F and F∩{k }=ϕ.Iff k∈G, h$\notin G\ and\ h \in F, k \notin F$, Hence $(\mathcal{K},\mathfrak{T})\ is\ \omega - T_1 space.$

Defintion3.15-:let $(\mathcal{K},\mathfrak{T})$be a topological space ,then $\mathcal{K}$is said to

1) R_0 space if ∀ open set Uand kϵU, then cl$\{\mathcal{K}\} \subset U$. [2,3,8,10].

2)ω -R_0space ,if ∀ ω-open set Uand kϵU then $cl_\omega\{\mathcal{K}\} \subset U$

3)$\omega_1 - R_O spaceif$ ∀ open set Uand k∈Uthen $cl_\omega\ \{\mathcal{K}\} \subset U$.

4)$\omega_2 - R_0 spaceif$ ∀ open set U and k∈U,then cl{$\mathcal{K}$ }⊂ ω -open set U

Thorem3.16:-For any topological space $\mathcal{K}$,the following statement equavelent :-

1)$\mathcal{K}\ is\ \omega - R_0\ space.$

2) For any non-empty set A and G $\epsilon\ \omega.O(\mathcal{K})$∋ A∩G≠$\phi$,∃ Z∈ ω -C $(\mathcal{K})$∋ A∩Z ≠ϕ and Z⊂G .
3) For any G∈ $\omega.O(\mathcal{K})$,G=∪ {Z ∈ ω –C $(\mathcal{K})$:Z⊂ G}

4) For any Z∈ ω.C $(\mathcal{K})$,Z =∩{G∈ $\omega.O(\mathcal{K})$:Z⊂ G}

5) For any k∈ $\mathcal{K}$, $cl_\omega\{\mathcal{K}\}$⊆ $\Lambda_\omega\{\mathcal{K}\}$. } .

Proof:-1→2 Let A be anon-empty subset of $\mathcal{K}$,and G∈ $\omega.O(\mathcal{K})$ ∋ A∩G≠ϕ. Let k∈A∩G, then $k \in G \in \omega.O(\mathcal{K})$ we have by (1)$cl_\omega\{k\}$⊂ G.Put Z=$cl_\omega\{k\}$, then Z∈ ω.C $(\mathcal{K})$,Z⊂ G and A∩ $Z \neq \phi$..

2→3 Let G∈ $\omega.O(\mathcal{K})$.Then G⊂∪ {Z∈ ω.C $(\mathcal{K})$:Z⊂ G

Let k∈ G, then ∃ Z∈ $\omega.C(\mathcal{K})$ ⊃kϵZ, and Z⊂G.Thus kϵZ ⊆∪ {Z∈ $\omega.C(\mathcal{K})$:Z⊆ G}Hence (3)follows.

3→4 it is trival. .

4→5 it is trival.

5→1 it is trival.

Thorem3.17:-Let $(\mathcal{K},\mathfrak{T})$be a topological space ,then $h \in \Lambda_\omega$ {k }if and only if k∈ $cl_\omega\{h\}$.

Proof:-⇔ $h \in \Lambda_\omega\{k\}$=∩{G∈ $\omega.O(\mathcal{K})$ }⇔ $h\epsilon G\ \forall G\ \in \omega - open\ set.$

⇔ $\omega - open\ set\ contaning\ h.$⇔ $k\ \epsilon\ cl_\omega\{h\}$.

Thorem3.18:-Let $(\mathcal{K},\mathfrak{T})$) be a topological space .then $(\mathcal{K},\mathfrak{T})$ is $\omega - R_0\ if\ and\ only\ if\ \ cL_\omega\{k\} = \Lambda_\omega\{k\}\ \ \forall\ k \in \mathcal{K}$

Proof :It is clear by using theorem 3.16,3.17.

Thorem3.19:-A topological space $(\mathcal{K},\mathfrak{T})$ is an $\omega - R_0$if and only if ∀Uopen set and k∈U,then $cl_\omega(\Lambda_\omega\{k\} \subseteq U$.

Proof:- It is clear by using theorem 3.17and Defintion3.15 part(2).

Theorem3.20:-A topological space $(\mathcal{K},\mathfrak{T})$ is an $\omega - R_0$ *if* and only if ∀ closed set Z and k∈ Z,then $\Lambda_\omega\{k\}\subset Z$.

Proof:-It is clear by using theorem3.17.

Defintion3.21:- Let $(\mathcal{K},\mathfrak{T})$ be a topological space ,$\mathcal{K}$ *is said to be*: $-$

1)R_1 *space if* ∀ *two distanct point k and h of* $\mathcal{K}$ *with* $cl\{k\} \neq cl\{h\}$. ∃*disjoint open set* $U,V \ni cl\{k\} \subseteq U,V$[2,3,8,10].

2)$\omega - R_1$ *space with* $cl_\omega\{k\} \neq cl_\omega\{h\}$*and* ∃ $\omega -$ *Open set* $U,V \ni cl_\omega\{k\} \subseteq U,V$.

3)$\omega_1 - R_1$ *sPace* with cl{k}$\neq cl\{h\}$ *and* ∃ $\omega -$ *open set* $U,V \ni cl\{k\} \subseteq U,V$.

4)$\omega_2 - R_1$ *space with* $cl_\omega\{k\} \neq cl_\omega\{h\}$ *and* ∃*open set* U,V.

Theorem3.22:-A topological space $(\mathcal{K},\mathfrak{T})$ is an $\omega - R_1$space if and only if for each k$\neq h \in \mathcal{K}$

With $cl_\omega\{k\} \neq cl_\omega\{h\}$, *then* ∃$\omega -$ *oPen set* $U,V \ni cl_\omega(\Lambda_\omega\{k\}) \subseteq U$*and*$cl_\omega(\Lambda_\omega\{k\}) \subseteq V$.

Proof:-It is clear by using theorem 3.18 and Remark 1.13[2].

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